Proper Generalized Decomposition Applied on a Rotating Electrical Machine

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The Proper Generalized Decomposition (PGD) is a model order reduction method which allows to reduce the computational time of a numerical problem by looking for a separated representation of the solution. The PGD has been already applied to study electrical machine but at standstill without accounting the motion of the rotor. In this communication, we propose a method to account for the rotation in the PGD model of an electrical machine.

***Index Terms*—Finite Element Method, Model Order Reduction, Proper Generalized Decomposition, Electrical Machine**

I. Introduction

APPLYING the Finite Element (FE) method to model electrical machines is now very common. This approach enables to obtain very accurate results but requires solving large scale systems, leading to a high computational cost. Moreover, when the equations of the model depend on a significant amount of parameters, the required number of solution of the FE model to efficiently characterize the problem explodes. In order to reduce the computational cost, model order reduction methods have been proposed. The two most common model order reduction methods which deal with parametric problems are the Reduced Basis (RB) and the Proper Generalized Decomposition (PGD). The RB approach consists in approximating the solution in a reduced basis, leading to a numerical problem with few unknowns. As for the PGD, the solution is approximated with a separated representation, allowing to efficiently deal with parametric problems. The PGD has already been successfully applied to model electric devices such as 3-phases transformers [1]. However, the PGD has not been used to model problems accounting for the motion of an electrical machine. In this communication, we apply the PGD to a 2D linear FE model of a rotating electrical machine.

II. FE magnetostatic problem of an electrical machine

Let us consider a magnetostatic problem of a 2D synchronous generator in a domain D of boundary Γ. The machine is composed of a rotor domain D_θ and a stator one $D\backslash D_\theta$. Four stranded inductors supplied by the currents $i_k,\ k=0,\dots,3$ are considered as shown in Fig.1. The linear magnetostatic vector potential formulation is given by:

$$\mathbf{curl}\left(\nu(\boldsymbol{x})\mathbf{curl}\boldsymbol{A}(\boldsymbol{x})\right)=\sum_{j=0}^{3} i_j \boldsymbol{N}_j(\boldsymbol{x}) \qquad (1)$$

with $\boldsymbol{A}$ the vector potential. $\boldsymbol{N}_j$ is the unit current density vector flowing through the j^{th} stranded inductor and i_j its associated current. $\nu(\boldsymbol{x})$ denotes the reluctivity of the linear isotropic materials. Moreover, the following boundary condition is applied on Γ: $\boldsymbol{A}\times\boldsymbol{n}=0|_\Gamma$. The FE model is obtained by approximating $\boldsymbol{A}$ with N nodal elements in 2D. Furthermore, the Overlapping Finite Element Method is used in order to take into account the motion of the rotor without any remeshing process [2]. Finally, the linear system of equations describing our problem reads:

$$\left(\boldsymbol{M}+\boldsymbol{M}_{Ovl}(\theta)\right)\boldsymbol{X}=\sum_{j=0}^{3}\boldsymbol{F}_j i_j \qquad (2)$$

with $\boldsymbol{X}$ the unknown vector whose k^{th} component is the value of $\boldsymbol{A}$ on the k^{th} node. $\boldsymbol{M}$ is the stiffness square matrix of size N which is symmetric positive definite while $\boldsymbol{M}_{Ovl}(\theta)$ denotes the Overlapping matrix accounting for the motion of the rotor after a rotation of angle θ. As for $\boldsymbol{F}_k,\ k=0,\dots,3$, they refer to the four vectors of size N depending on the unitary current density $\boldsymbol{N}_j$.

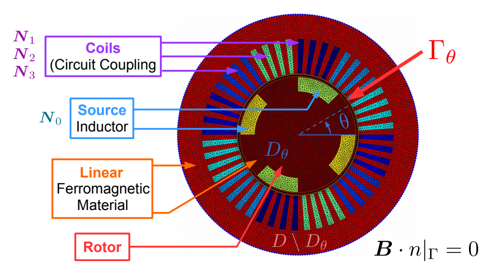

Fig. 1. Electrical machine

III. Model Order Reduction with the Proper Generalized Decomposition

Even though fine results are obtained by solving the FE problem (2), the associated computational time can be prohibitive. For instance, when the FE system is called within an optimization loop or coupled to an electrical network, fast evaluations are required. In order to achieve accuracy with an acceptable computational cost, the Proper Generalized Decomposition [3] can be used.

A. Proper Generalized Decomposition

The PGD is an iterative method enabling to approximate the solution of a parametric problem, by iteratively constructing an approximation in a separated representation: the solution is written as a sum of products of functions which depend on single parameter. Let $g(\mu_1, \ldots, \mu_p)$ be the solution of an equation depending on p parameters: $\mathcal{H}(g, \mu_1, \ldots, \mu_p) = 0$. The PGD approximation g^m reads at the m^{th} iteration:

$$g^m(\mu_1, \ldots, \mu_p) = \sum_{k=1}^{m} f_k^{\mu_1}(\mu_1) f_k^{\mu_2}(\mu_2) \ldots f_k^{\mu_p}(\mu_p) \quad (3)$$

$$= g^{m-1}(\mu_1, \ldots, \mu_p) + f_m^{\mu_1}(\mu_1) \ldots f_m^{\mu_p}(\mu_p) \quad (4)$$

The last equation highlights the iterative process of the PGD: only $f_m^{\mu_1}(\mu_1), \ldots, f_m^{\mu_p}(\mu_p)$ have to be determined at the m^{th} iteration of the PGD, $g^{m-1}(\mu_1, \ldots, \mu_p)$ being previously determined. To compute these p functions, the residue of the equation induced by the approximation g^m: $Res(g^m) = \mathcal{H}(g^m, \mu_1, \ldots, \mu_p)$ is expressed. Then, $f_m^{\mu_k}(\mu_k),\ k = 1 \ldots p$ are the solutions of the following equations which depends on a single parameter:

$$(Res(g^m), \prod_{j \neq k} f_m^{\mu_j}(\mu_j)) = 0, \quad \forall k \in [1, p] \quad (5)$$

where $(\cdot, \cdot)$ denotes a scalar product on $(p-1)$ parameter. Finally, a fixed point algorithm enables to find the unknown functions $f_m^{\mu_k}(\mu_k),\ k = 1 \ldots p$ by iterating between the p equations (5) [3].

B. Application of the Proper Generalized Decomposition on the electrical machine equation

To efficiently apply the PGD to our problem, several steps must be carried out to reformulate the original problem and the separated representation of the solution.

1) Affine decomposition of the equation

The equation on which the PGD is applied should have an affine decomposition. Thus, the operators (right-hand-sides) are rewritten as a sum of products of operators (right-hand-sides) which only depends on a single parameter [3]. In our problem, equation (2) has not an affine decomposition since $\boldsymbol{M}_{Ovl}(\theta)$ is a matrix (accounting for the spatial parameter) depending on θ (angular parameter). To deal with this problem $\boldsymbol{M}_{Ovl}(\theta)$ is precomputed for l discrete values θ_l in $[0, 2\pi]$: $\boldsymbol{M}_{Ovl}^k = \boldsymbol{M}_{Ovl}((k-1)\Delta\theta),\ k = 1, \ldots, l$ with $\Delta\theta = 2\pi/l$. Then, the Overlapping operator is approximated in the following affine decomposition:

$$\boldsymbol{M}_{Ovl}(\theta) \approx \sum_{k=1}^{l} \boldsymbol{M}_{Ovl}^k \alpha^k(\theta) \quad (6)$$

with $\alpha^k(\theta) = 1$ if $\theta \in [(k-1)\Delta\theta, k\Delta\theta[$ and is null elsewhere.
Remark: Even though the angular grid is quite fine, computing the Overlapping matrices $\boldsymbol{M}_{Ovl}^k$ is cheap in terms of time and memory space since they are sparse matrices restricted to nodes located along the sliding domain on which is computed the motion.

2) Enforcing the superposition principle in the separated representation

By directly applying the separated representation presented in the equation (3), the PGD algorithm struggles in converging to an accurate approximation. For instance, the separated representation coupled with the fixed point algorithm does not naturally capture the fact that the solution is null if $i_0 = i_1 = i_2 = i_3 = 0$. This fact could have been expected since the univariate functions $f_k^{i_0}(i_0)$ are not null for $i_0 = 0$ (otherwise the solution would be null for $i_0 = 0$ and $i_1 = 1$ for example). To tackle this problem, the solution is approximated by enforcing the superposition principle as:

$$\boldsymbol{X}(\theta, i_0, i_1, i_2, i_3) = i_0 \boldsymbol{X}^0(\theta) + i_1 \boldsymbol{X}^1(\theta) + i_2 \boldsymbol{X}^2(\theta) + i_3 \boldsymbol{X}^3(\theta)$$

where $\boldsymbol{X}^k(\theta)$ is the PGD approximation of the problem (2) with $i_k = 1$ and $i_j = 0,\ j \neq k$.

IV. Application

The 2D mesh of the electrical machine is presented on Fig. 1 (17248 elements and 8913 nodes). The angular grid is discretized on $l = 50$ points in $[0, 2\pi]$. Fig. 2 presents the errors associated with the fluxes flowing through the 3 inductors located in the stator versus the number of iterations of the PGD approximation $\boldsymbol{X}^0(\theta)$, defined in (III-B2). Fig. 3 shows the same fluxes versus the position of the rotor, computed with both the FE code (with $i_0 = 1$ and $i_1 = i_2 = i_3 = 0$) and the PGD approximation $\boldsymbol{X}^0(\theta)$ with 30 iterations. The waveforms obtained with the PGD match the one from the FE code.

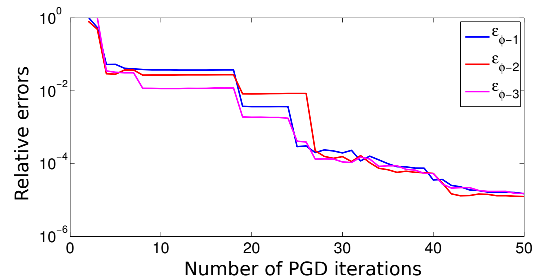

Fig. 2. Errors associated with the fluxes in the stator inductor

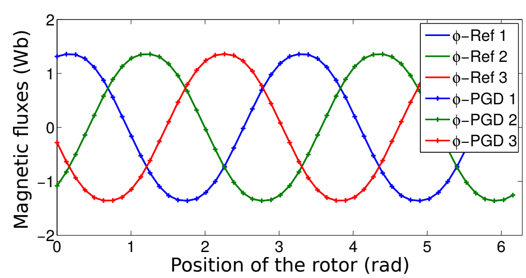

Fig. 3. Magnetic fluxes in the stator inductors

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